THE

Quadrature of the Circle,

In Two General Methods,

CLEARLY DEMONSTRATED.

THE QUADRATURE OF THE CIRCLE,

In Two General Methods,

CLEARLY DEMONSTRATED.

METHOD I.

Is founded on that general Theorem, that all Figures ought to be meaſured by their Similars; therefore Circles are to be meaſured by Circles.

METHOD II.

Is founded on that known Definition, that if a Circle apply every Part of its Circumference one after another to a ſtreight Line, that Line will equal its Circumference.

ALSO

The laying down the Cycloid geometrically by Points: And the Triſection of Angles geometrically performed.

To which is annexed,

A new and eaſy Method of gaining the Longitude at Sea.

By JAMES LATIMER.

NEWCASTLE:

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MDCCLXXV.

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TO

GOD,

THE one eternal, the incomprehensible, the omnipresent, omniscient, and almighty Creator of all things that exist, who without scale or compasses, divided infinite space into proportional parts, and gave those innumerable systems of the universe their limited space to act in, this Atom of Geometry is dedicated and devoted, with all possible gratitude, humiliation, worship, and the highest adoration both of body and mind, by

His most resigned, low,

And humble creature,

JAMES LATIMER.

RECOMMENDATIONS *by the following* GENTLEMEN.

Mr Latimer, *Newcaſtle*, *April* 20*th*,

I Have peruſed this treatiſe, and I think it is not only the beſt done thing of its kind, but ſo far as I am able to judge, a ſtreight line equal to the circumference of the circle, is truly and geometrically determined, and conſequently all other things that depend thereupon. ROBERT TURNBULL,

TEACHER *of the* MATHEMATICS.

Mr Latimer, *Newcaſtle*, *May* 9*th*, 1774.

I Have peruſed your method of finding the Quadrature of the Circle, and that of finding the Longitude at Sea:—As I have only had time to examine it ſlightly, cannot ſay ſo much for it as I might do on a more ſtrict inſpection, but think it merits the encouragement of the public, and heartily wiſh you ſucceſs in your intended publication. I am, Sir, *Your humble ſervant*,

J. FRYER, *Mathematician*.

SIR, *Howdon Pans*, 21*ſt April*, 1774.

HAVING peruſed your ſcheme for the geometrical Quadrature of the Circle, or to find a ſtreight line equal in length to any portion of the arch thereof, is not only exactly done, but alſo ſo plainly demonſtrated, in my opinion, upon geometrical principles, that while it heightens the eſteem of the performance, in the minds of the truly candid

did Lovers of Science, it will at once, I hope, ſtop the mouths of thoſe Argus's, who are all eyes to ſearch for faults, in order to condemn every thing that is not of their own production. I heartily approve of your ſchemes for Longitude and Triſection, and, wiſhing a kind acceptance of your well-meant endeavours, I remain, dear Sir,

Your humble ſervant and fond admirer,

JOHN MILNE.

Mr Latimer, *Gateſhead, May,* 26, 1774.

HAVING peruſed your hitherto incomparable treatiſe, upon the Quadrature of the Circle, &c. &c. my opinion is, that it is not only a noble, but a very uſeful performance and cannot fail (eſpecially by all the ſtudious ſons of Matheſis) meeting with a reception ſuitable to its exalted merit; as by your unerring demonſtrations, you have fully and perſpicuouſly determined every thing which depends upon a geometrical definition.

Go on, great Sir, Zoilians to ſubdue,
And all opponents unto maxims true:
Extend this ſcience round the ſpacious earth,
That future ages may admire your worth;
Let Momus jeſt, and all illit'rate foes,
You'll be rever'd, nor rank'd with none of thoſe.

ALEX. MURRAY,

Teacher of the Mathematics.

MR Lowthion's Compliments to Mr Latimer;—has peruſed his Treatiſe upon the Quadrature of the Circle, &c. and thinks he deſerves encouragement for his labour and ingenuity.

PREFACE.

THE Quardrature of the circle has been of ſo engaging a nature, as to employ the minds of the moſt eminent geometricians in all ages. Even from the moſt ancient times they have exerciſed their wits upon it; and many more had done the ſame, if they had not ſeen their pains (though undertaken for the common good, although not brought to perfection) vilified by thoſe who envy the praiſes of other men. Amongſt thoſe ancient writers Archimedis was the only man that ever ſtruck boldly at finding a ſtreight line equal to the circumference of a circle: and his definition, of a circle applying its circumference by rolling along a ſtreight line, and applying every part of its circumference, one after another, to the ſtreight line, until the point that was in contact with the ſtreight line at firſt, comes in contact with it again; then that ſtreight line will be equal to the circumference of that circle, is true (and indeed it is what I have built my whole foundation upon): but the finding of ſuch a ſtreight line geometrically, is what he could not effect, and many have attempted it in vain.

I ſhall mention ſome of them; and firſt Lalover, the jeſuit, in his attempt, ſays, that

there is equality between a ſtreight line and a crooked one; but now, ſays he, ſince the fall of Adam, it is not to be had without the ſpecial help of divine grace. Wherein I obſerve, that he ſuppoſing he had found the quadrature, would have us believe it was not by the ordinary and natural help of God, whereby one man reaſoneth, judgeth, and remembereth better than another; but by the ſupernatural help of God given to him of the order of Jeſus, above others that had attempted the ſame in vain.

Joſeph Scaliger and Mr Hobbs both attempted the quadrature of the circle, but without ſucceſs. Mr Hobbs indeed hath laid down no premiſes, and therefore no concluſion can be drawn from it, as may be ſeen in his Philoſophical Works, page 213.

Deſcartes, through great pride and preſumption, boldly affirming that there is no equality between a ſtreight line and a circle, and (contrary to common ſenſe Archimedes) has pronounced it impoſſible, and therefore has locked up the brains of our great modern mathematicians from attempting it in a geometrical way. Of ſuch great weight is the authority of one man to put out the eyes of poſterity for ages, becauſe he was not eagle-eyed enough to diſcover it. He therefore endeavoured to put a lock and key upon it, which indeed hath ſucceeded very well hitherto; but how unbecoming a man it is, to pronounce a thing impoſſible, that has

ſo

ſo true and clear a definition to found the diſcovery of that problem upon: it can proceed from nothing but pride and preſumption grown into madneſs.

Was not the 47th of Euclid 1. looked upon as impoſſible before Pythagoras found it? and was not the proportion of the cylinder, cone, and ſphere, before Archimedes' days, looked upon in the ſame light? And had theſe problems been undiſcovered in Deſcartes' days, I am perſuaded the world would never have ſeen them through him, for as great a name as he got. But let us leave Deſcartes, for we have plenty of them at home, without going to France, who can make great diſcoveries over a hot bowl of punch; and pronounce the word *impoſſible*, with as much facility and eaſe, as they can ſwallow their favourite liquor.

I am perſuaded, when Pythagoras found out the 47th of Euclid 1. that he was ſatisfied of the truth and certainty of the demonſtration, without having the approbation of an academy; and is ſaid to have offered an hundred oxen to the gods, as a tribute of thanks for ſo great a diſcovery. And I am as certain of the truth of finding a ſtreight line equal to the circumference of a circle, as he was of the 47th of Euclid 1. as will be proved by the demonſtration, without it fall into the hands of ſceptics: but I hope it is diſcovered in a more happy epocha, where it will meet with men of virtuous minds; lovers of truth, for truth'

taκe. But if they ſhould be of a proud, preſumptuous, arrogant mind, I challenge them to invalidate it; but at the ſame time to deliver their objections in writing, which will be doing themſelves great honour; for by laying hold of ſuch an opportunity, they will get themſelves a name, although they may not have a head for the diſcovery of unknown truths.

This arduous and knotty problem, which has baffled the ſagacity of mankind theſe two thouſand years, I am ſatisfied is clearly diſcovered, if it be admitted that there is any certainty in geometry.

Alſo the triſection of angles, right, obtuſe, and acute, are made as clear and certain as biſection, and that by an eaſy and ſhort demonſtration. I have in the quadrature made uſe of nothing but four definitions and one axiom, which are the true principles of geometry; and therefore if not ſound, may eaſily be detected.

THE QUADRATURE OF TH CIRCLE.

AXIOMS,

1. EVERY whole is greater than its part.
2. All the parts taken together, are equal to the whole.
3. Two things that are each equal to a third, are equal between themſelves.
4. Equal quantities added to equal quantities, their ſums will be equal.
5. Equal quantities taken from equal quantities, the remainders will be equal.

THEOREM I.

All ſimilar figures are to one another as the ſquares of their homologous ſides.

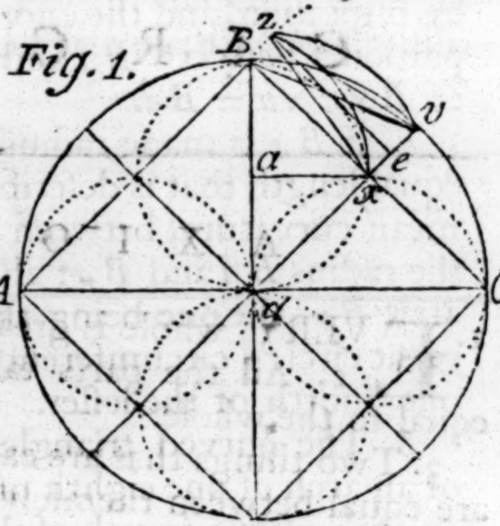
THEOREM II.

All circles are ſimilar, and are therefore to one another as the ſquare of their diameters.

All figures ought to be meaſured by their ſimilars (i. e.) a line by a line, a ſurface by a ſurface of the ſame kind; therefore circles are to be meaſured by circles.

DEFINITIONS *on Figure* I.

1. The diameter (*AC*) is double the diameter (*c d*), and the circumference of any of the leſſer circles is only half the circumfence of the greater.



2. The chord *BC*, is double the chord of any of the leſſer circles.

3. Double diameters give quadruple contents, the areas of the four leſſer circles taken together are equal to the area of the circumſcribing circle (pr theor. 2.) and every ſegment of the great circle, is equal to four of the leſſer ſegments by the ſame Theorem.

4. The greater ſegment *BvCD* is equal to four of the leſſer, then take away the ſegments *Bx* and *Cx*, and the remaining figure *BxCvB* is equal to what is taken away. *(per Def. 3d.)*

Let

5. Let the curvelineal figure $B\,x\,C\,v$ be biſected by the verſed ſine xv, and the curved triangles $B\,x\,v$, and $C\,x\,v$, will be equal to the ſegments $B\,x$, and $C\,x$, (by Def. 3) Alſo if a ſtreight-lined triangle can be proved equal to the curved triangle $B\,x\,v$ it will equal the ſegment $B\,x$, which done the quadrature of the circle, may be eaſily effected for four ſuch triangles being added to the inſcribed ſquare of one of the leſſer circles will equal the area thereof.

6. The radius $B\,d$ is double the radius $B\,a$ by inſpection, and the chord $B\,x$ is a mean proportional betwixt $B\,d$ and $B\,a$; for $B\,d : B\,x :: B\,x : x\,a = B\,a$.

7. If $B\,x$ is made radius, then every arch of equal length that is deſcribed by it, will have a mean curvature, between an arch deſcribed by the radius $B\,d$ and $B\,a$; alſo the arch $B\,v =$ the arch $B\,x$, the one being the one eighth of the great circle's circumference and the other the one fourth of the leſſer.

8. The curved triangle $B\,x\,v\,B$ is made up of an arch of one eighth of the great circle, and an arch of one fourth of the leſſer circle; therefore the arches are equal in length, but extreme in their curvatures, for their curvatures are as their radius which are extreme (*per fig.*) Now in order to reduce them to equal curvatures, let us conceive what is very eaſy: let the ſegment $B\,x$ move back upon the point x to z; at the ſame time let the arch $B\,v$ ſlide down the chord $B\,v$, until $B\,v$ equal $B\,x$, then will the chord $Z\,v$ equal the chord $z\,x$, and the arch $z\,v$ equal $B\,v$ in curvature, and length alſo.

And

And by this means we gain the true perpendicular height of the mixt triangle $B x v B$ by reducing the arches to equal curvatures; and $B z$ will equal one half $x v$, and $z e$ will be the true perpendicular.

LAMMA *Fig.* II.

Let there be drawn three circles Bd, Bx, and $B a$; whofe radii are $B d$, $B x$ and $B a$ (fig. 1ft.) by thefe radii the three circles, areas will be in : : (i e) as 1, 2, 4, and the circumference of the circle $B x$ will be a mean proportional be twixt the circumferences of the circles $B d$ and $B a$; alfo the chords and verfed fines are allproportionals and the verfed fine $d v$ is a mean between $x v$ and $a v$.

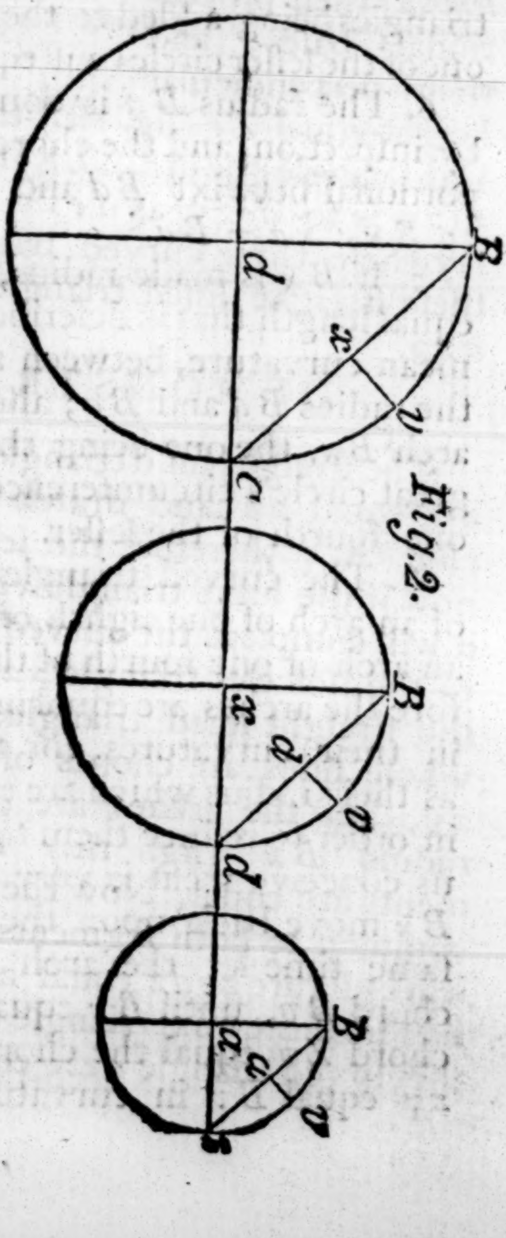
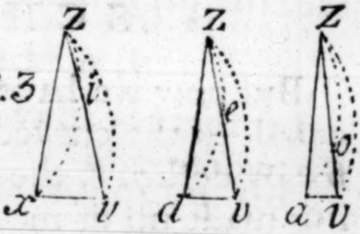


Fig. 2.

THEOREM *Fig.* III.

If three Isosceler curved triangles be drawn by the radii $B\,d$, $B\,x$ and $B\,a$, upon the versed sines of the three circles (*per lemma*) their proportional bases; and their perpendicular

Fig. 3



height equal $Z\,e$ (fig. 1st.) then will these curved triangles be in :: (*i. e.*) the curved triangle $x\,i\,z\,v$ is to $d\,e\,z\,v$ as $d\,e\,z\,v$ is to $a\,o\,z\,v$, for I say these three curved triangles are equal to these streight lined triangles, $x\,z\,v$, $d\,z\,v$, and $a\,z\,v$.

DEMONSTRATION.

The streight lin'd triangle $z\,x\,v$, is an isosceles $x\,z$ equal $z\,v$ and drawn by the radius $B\,x$ (fig. 1st. ; therefore the segments are equal; now if the mixt triangle $x\,i\,v$ be added to both, it will compleat the curved triangle $x\,i\,z\,v$ equal to the isosceles triangle $x\,z\,v$, (per axiom 4th,) the streight lined triangle $d\,z\,v$ is an isosceles whose sides are chords of the arches $z\,x$ and $z\,v$, and the arches are drawn by the mean radius $B\,x$ (fig. 1st.) therefore the segments are equal. Now the little segment $z\,e$ is common to both segments, which taken away leaves figure $z\,e\,d\,z$ equal $z\,e\,v\,z$, and if to both be added the mixt triangle $d\,e\,v$, it will compleat the curved triangle $d\,e\,z\,v$ equal the streight lined

tr iangle

triangle *z v d* (per axiom 4th and 5th), *Q. E. D.*

The ſame demonſtration will prove the curved triangle *z o a v z* equal to the iſoſceles triangle *z v a z.*

SCHOLIUM.

But now we have got a ſtrait lined triangle equal to the curved triangle *B x v B* (fig. 1ſt.) for by def. 7. 8. the curved triangle *z e d v z* being made up of mean proportionals, is equal to the curved triangle *B x v B*, (fig. 1ſt.) being made up of extreams by Def. 5. But by the laſt demonſtration the curved triangle *z e d v* equal the ſtreight lined triangle *z v d* therefore the ſtreight lined triangle will be equal the curved triangle *B x v B*. Alſo by axiom 3d.

From hence it will be eaſy to make a ſquare whoſe area ſhall be equal to the area of any given circle; for if four ſuch ſtreight lined triangles be added to the inſcribed ſquare (according to Euclid) that ſquare will = the circle ſo given; for theſe four triangles will equal the four ſegments of any of the leſſer circles (*per def. 4th. axiom 4th.*)

This demonſtration is founded upon the moſt ſelf-evident truths that geometry is capable of; therefore if it is not admitted, geometry muſt be an abſurd, vague, empty ſcience, wherein there is no truth.

PROPOSITION,

Any circle given, as the circle *L e N G t H*, to find a ſtraight line equal to the circumference.

DEFI-

DEFINITIONS *on Fig.* IV.

1. If the circle *LeNGtH* roll along the ſtreight line *LH*, applying every part of its circumference one after another unto the ſtreight line until the point *L* comes in contact with the line *LH*, which will be at *H*, then I ſay, that ſtreight line will be equal to the circumference of the generating circle *LeNGtH*.

2. If the quadrantal arch *LeN* be applied to every point of the arch one after another to the ſtreight line *LN*, then will that ſtreight line equal the quadrantal arch *LeN*.

3. When the quadrantal arch has applied every point of it to the ſtreight line, then will the point *L* be in contact with a ſtreight line drawn thro' the center of the generating circle *CB*, parallel to the indefinite line *DE*.

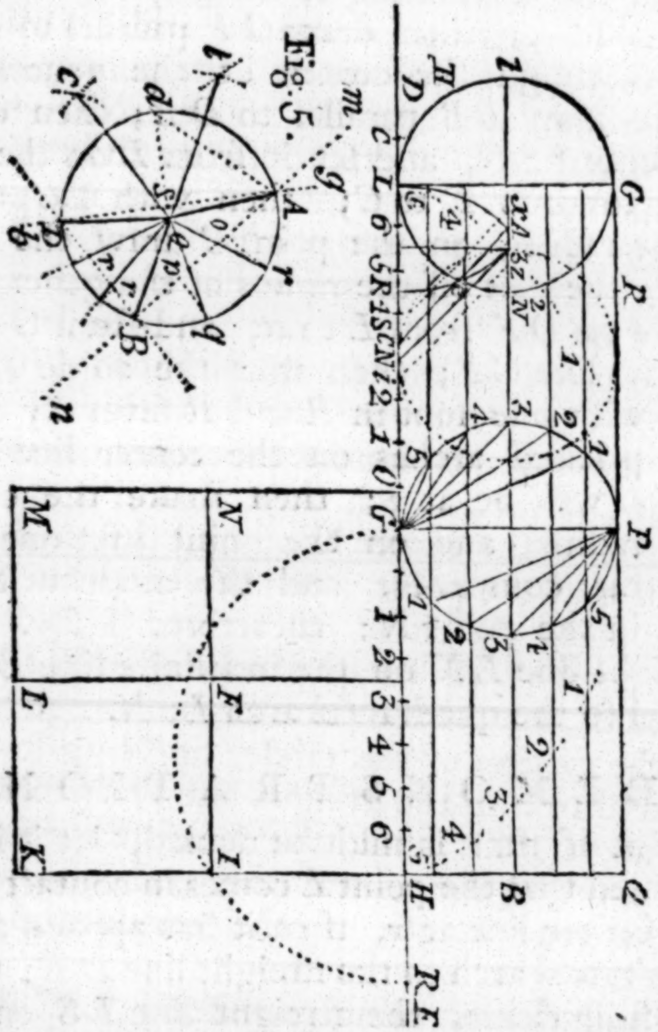
4. N. B. Let us conceive the generating circle applying its quadrantal arch *LeN* to the ſtreight line *DE*, and at the ſame time carrying the chord *LN*, and the angle *ACL* with it, being faſt to to the circle at the point *L*, but looſe and detached from it in all other parts ſo that it may be carried by the point *L* only: I ſay that when the point *L* comes in contact with the center line *BC* of the generating circle; that the angle *ACL* will be turned about by the application of the quadrantal arch to the ſtreight line; and the point *A* will fall upon the point *R* on the line *DE* and the point *L*, will be in contact with the center line at 3, and the chord *LN*, ſet off from 3, will touch

touch the line *D E* in *N*, and *L N* on the line *D E* will equal the quadrantal arch; for its by the angle *ACL* that we find the point 3 on the center line, and then the chord *L N* from that point finds the length of the quadrantal arch in the line *D E*; becauſe the generating circle for the laying down the quadrantal arch, can never exceed that angle from radius laid down on the line *D E*; nor can it be leſs.

To prove which, ſet off the chord *L N* from *R* on the indefinite line towards *D* to *C* and making the chord *L N* radius, upon the point *C* draw an arch from *R* and it will paſs thro' 3 on the center line; and this is a clear proof that the point 3 on the center line is the very point the circle has arrived to, when it has applied all its quadarantal arch to the line *D E*, for its the ſame angle, turned about by the motion of the circle, applying its quadrantal arch to the ſtreight line *D E*; therefore the angle *ACL* is the true meaſure for finding the point on the central line, and conſequently the true length of the quadrantal arch, which is what is wanted.

Theſe things being premiſed I ſhall aſſume it as a problem, and the manner and method of finding a ſtreight line equal to the circumference of any given circle is as followeth.

PROB.



PROBLEM.

ANY circle, as *LeNGtH* given, to find a streight line equal to the circumference, and consequently making a square whose area shall be equal to the area of the circle.

First draw an indefinite line *DE* to touch the circle in the point *L*, and *LG* the diame-

 ter

ter of the generating circle perpendicular to the point *L*; then draw *G P* parallel to *D E*, alſo through the center of the generating circle draw *C B* parallel to *D E*; then draw the chord *L N*, and ſet it from *L* on the line *D E* towards *E* to *C*; then with *LC* = *L N* made radius, on the point *C* draw the arch *L A*, then ſet off the radius of the generating circle on *D E* from *L* to *R*, and from *G* to *R* on the line *G P*; then take the angle *L C A*, and with one foot in *R* and *R* ſeverally draw the pricked arches on the center line *C B*, which will be at 3; then make the chord *L N* radius, and on the point 3 ſet one foot of your compaſſes, and the other will cut *D E* in the point *N*: therefore, I ſay, that ſtreight line *L N* on the indefinite line *D E* is equal to the quadrantal arch *L e N*.

DEMONSTRATION.

But if that ſhould be denied, let it it be granted that the point *L* comes in contact with the center line at *x*, then it has applied all its quadrantal arch to the ſtreight line *D E*; then, by ſuppoſition, the ſtreight line *L S* on the line *D E* would be equal to the quadrantal arch: but, I ſay, by applying of the angle *L C A* from *R*, equal the radius of the generating circle laid on the line *D E*, that it will not reach the point *x*; which proves, by definition 4th, the circle had not applied all its quadrantal arch to the ſtreight line *D E*, by the ſuppoſition of *x* being the point; neither can there be any point taken between *A* and

3 that

3 that the angle *A C L* will reach, and therefore the ſuppoſition of *x* is abſurd. But let it be granted that the circle, by applying its quadrantal arch to the ſtreight line *D E*, may have brought the point *L* in contact with the center line *C B* at *z*, I ſay, that the angle *L C A* applied from radius laid on the infinite line *D E* (*i. e.*) from *R* it will be found to reach above *z* on the center line *C B*, therefore the ſuppoſition of *z* is abſurd alſo; and as there cannot be any point taken between *A* and 3 on the center line *C B* but what the application of the angle *L C A* will ſhow, that the circle has not applied all its quadrantal arch to the ſtreight line *D E*, ſo neither can there be any point taken between 3 and *N* but what the application of the angle *L C A* will ſhow, that the circle muſt have applied more than its quadrantal arch to the line *D E*, by definition 2d, 3d, and 4th: therefore the ſtreight line *L N* on the line *D E* is equal to the quadrantal arch of the generating circle *L e N*. *Q. E. D.*

But it is evident by the laſt demonſtration, that when the circle has applied all its quadrantal arch to the ſtreight line *D E*, that the poſition of the circle will be repreſented by the pricked circle; and it is alſo evident, that if the pricked circle was to apply its quadrantal arch (*i. e.*) from *N* on the line *e* 3, to the line *D E* backwards to *D*, the point *L* which is come in contact with the center line *C B* at 3, muſt, by ſuch application, recede to the

point *L* on the line *DE*, and be in the ſame poſition it was in before it ſet out for the firſt application (*i.e.*) it will be in the ſame poſition with circle *LeNGtH*; then will the point *N* on the line *DE* be advanced to *N* on the center line *CB*, and the point 3 deſcended to *L* on the line *DE*.

Alſo the point *N* on the center line *CB* may be found where the pricked circle, by applying its quadrantal arch to the ſtreight line *DE* backwards to *D*: thus, by the angle *LCA* being applied from *tr* on the line *DE*, ſet off from *N* towards *D*, which ſignifies radius, and it will find the point *N* on the center line *CB*.

But ſet off *LN* from *N* toward *G*, to *t*, and from *t* to *H* on the line *DE*. I ſay, the ſtreight line *LeNGtH* is equal to the circumference of the generating circle *LeNGtH*, becauſe *LN* equal the fourth of the circumference by demonſtration, and the ſtreight line *LeNGtH* equal to 4 times *LN*; therefore it muſt equal the whole circumference of the generating circle, becauſe all the parts taken together are equal to the whole. *Q.E.D.*

But the parallelogram *GCBH*, being the rectangle of radius into the ſemi-circumference, is equal to the area of the generating circle *LeNGtH*, and the line *HI* being a mean proportional between $HB=HR$, and the ſemi-circumference *GH*; therefore the ſquare *HF* equal the area of the generating circle, by Euc. 6th and 14th. *Q.E.D.*

SCHO-

SCHOLIUM.

I have given a double demonſtration to this arduous and knotty problem, becauſe the ſolution of it has baffled the ſagacity of Geometricians for above theſe two thouſand years without any ſucceſs, and many have maintained it impoſſible; Deſcartes in particular inſiſts on it, that a ſtreight line and a circle being of different natures, there can be no ſtrict proportion between them; but any one that ſeriouſly conſiders (def. of fig. 4th) muſt conſequently grant there is ſome ſtreight line equal to the circumference of a circle; and, by conſequence, ſince there is ſome ſtreight line equal to the circumference, the difficulty only lies in laying down ſuch juſt premiſes as could naturally lead us to a juſt compariſon, and that is effected by drawing the ſtreight line through the center of the generating circle, parallel to the indefinite line *DE*; and in conceiving of the circle, applying its quadrantal arch, and carring the angle along with it, which finds the point on the center line when the circle has juſt laid down all its quadrantal arch.

COROLLARIES.

Firſt, from this problem it will be eaſy to conceive how to make a ſquare equal to the curve ſuperficies of any cylinder; for if the height of the cylinder equal the diameter of its baſe, then will the parallelogram *LGQH*, fig. 4th, equal the curve ſuperficies of the cy-

 linder,

linder, being the rectangle of the height into the circumference of its base; and the parallelogram $HINO$ equal the area of the two bases by the last demonstration; therefore, the parallelogram $LGQH+HINO$, equal the whole superficies of the cylinder, which is easily reduced to a square by Euc. 6th and 14th.

Second, a sphere or globe being equal to four times its greatest circle, therefore if the circle $LeNGtH$ equal the greatest, then the square HM being equal to four times HF, it will be equal the curve superficies of such a sphere, by the last demonstration.

The superficies of a cone may be squared after the same manner.

SCHOLIUM.

These problems of making squares equal to the superficies of cylinders, cones and spheres, depending on problem 1st (*i.e.*) of finding a streight line equal to the circumference of a given circle, was thought entirely impossible; but now may be very easily effected, and much more added to the science of geometry, such as the finding of streight lines equal to the curve lines of all the conic sections, *viz.* ellipses, parabola, and hyperbola, which will open a new field of discoveries, and bring geometry to the utmost perfection.

Charles the fifth, Emperor of Germany, offered a reward of one hundred thousand crowns to any person who should solve this celebrated problem; and the States of Holland have

have propoſed a reward for the ſame, but I know not what it is.

The laying down of the CYCLOID *geometrically by* POINTS.

I Shall now proceed to ſhow how to lay down the cycloid geometrically by points; for hitherto it hath only been a mechanical line, and never could be brought within the ſcience of geometry: But in order to lay it down, it will be neceſſary to premiſe the following definitions.

Firſt, the periphery of the cycloid is generated by the motion of the point *L* ſliding through the points 5, 4, 3, 2, 1, *P*, and *P*, 1, 2, 3, 4, 5, *H*, while the generating circle is applying its circumference to the indefinite ſtreight line *DE*.

The firſt half of the cycloid *L*, 5, 4, 3, 2, 1, *P*, is generated by an indefinite number of chords of the generating circle iſſuing out from the point *L*, the greateſt of which is the diameter of the generating circle. The other half of the cycloid *P*, 1, 2, 3, 4, 5, *H*, is generated by an indefinite number of chords decreaſing in the point *L*, paſſing through the points 1, 2, 3, 4, 5, *H*; and the baſe of the cycloid is the circumference of the generating circle *LeNGtH*, as may be ſeen by the conſtruction of fig. 4th.

Now to lay down the cycloid, ſet off *LN* along the line *DE*, from *N* to *G*, and draw

GP

GP parallel to *LG*, and upon *GP* describe the generating circle, and bisect the arches 3, *P*, and *P*, 3, at 1 and 5; and also bisect the arches 3, *G*, and *G*, 3, below the center, which will be at 5 and 1 *per* fig. then bisect the arches 3, 1 and 3, 5, and the arches 3, 5, and 3, 1 below the center; then draw their corresponding chords *G* 1, *G* 2, *G* 3, *G* 4, *G* 5, and *P* 5, *P* 4, *P* 3, *P* 2, *P* 1; then draw lines parallel to the base of the cycloid through the points 1 5, 2 4, 3 3, 4 2, and 5 1; then bisect the line *NG* on the base of the cycloid, which will be at 1; also bisect *G t*, which equal *NG*, at 1; then bisect *N* 1 at 2, and *t* 1 at 2 on the other side *G*; then take the corresponding chords of the generating circle, and on the points 1 and 2 on the base of the cycloid cut the coresponding parallels which pass through the points of the chords 1, 5, 2, 4, which will cut the parallels in the points 1, 1 2, 2; also take the line *L* 1 and set off on the parallel 5 1 below the center from 5 to 5, and also on the other side on the same parallel from 1 to 5; then take *L* 2 on the base and set off upon the 2d parallel from 4 to 4, and on the other side from 2 to 4; I say, that the points *L*, 5, 4, 3, 2, 1, *P* and *P*, 1, 2, 3, 4, 5, *H*, are all points in the curve of the cycloid.

DEMONSTRATION.

The property of the cycloid is the sine of the angle *PC* 3 + the arch *P* 1 2 3, equal the ordinate *C* 3 3; for *G* 3 being parallel to *N* 3, and

and GN parallel to 3 3; therefore the line 3 3 equal GN, but GN equal the quadrantal arch by demonſtration; for the point L when the circle has applied all its quadrantal arch to the ſtreight line DE, is come in contact with the center line CB, therefore the point 3 is a point in the curve of the cycloid. *Q.E.D.*

This central ordinate, being found by demonſtration, may be called the LATUS RECTUM of the cycloid, by which all the ordinates of the cycloid may be regulated and found; for that being the common property of the cycloid, as has been proved, then it will be always the ſine of the arch of the generating circle + the arch itſelf, equal the correſponding ordinate of the cycloid (*i. e.*) the line L 1 on the baſe is equal the arch G, 5, 4, 3, 2, 1, by the property thereof; therefore the correſponding chord G 1 will reach from 1 on the baſe to the point 1 on the 2d parallel above the center; and becauſe the line L 1 on the baſe equal the arch G, 5, 4, 3, 2, 1, of the generating circle, therefore the point L will be arrived to the point 1 on the 2d parallel, when it has laid down an arch equal to the ſtreight line L 1, therefore the point 1 on the 2d parallel above the center line will alſo be a point in the curve of the cycloid. And after the ſame manner the chord G 2 will be found, to reach and cut the firſt parallel at 2 above N: And therefore the point 2 is another point in the curve of the cycloid.

And if L 1 on the baſe be added to the ſine a 5, *i. e.* ſet off from 5 to 5 above L, this point

5 will

5 will be in the curve of the cycloid alſo by the above property. And if the line *L* 2 be added to the ſine *b*, 4, it will find the point 4 on the parallel above *L*, which will be another point in the curve of the cycloid.

Alſo becauſe the ſtreight line *L* 1 equal the arches *G* 5 + 5 4 + 4 3 + 3 2 + 2 1, and added to the ſine *a* 5 compleats the ordinate of the cycloid *a* 5 5; and the line *L* 2 = the arches *G* 5 + 5 4 + 4 3 + 3 2; and added to the ſine *b* 4, it will compleat the ordinate of the cycloid *b* 4 4: Then by the property of the cycloid theſe points ſo found will all be points in the curve of the cycloid; and if the points of the curve of the cycloid on the other ſide of *G P*, be found after the ſame manner, and the curve line be drawn through theſe points, it will compleat the whole curve of the cycloid *L*, 5, 4, 3, 2, 1, *P* + *P*, 1, 2, 3, 4, 5, *H*. *Q. E. D.*

But if theſe arches were biſected again and their correſponding chords drawn, and likewiſe the ſtreight lines that equal the arches on the baſe biſected, and the correſponding parallels drawn through the points of the chords, then there might be 27 points found in the curve of the cycloid, by which it might be delineated more exactly; for the more points there are found, the more exactly the curve will be drawn.

This figure of the cycloid hath hitherto been a mechanical one, and never could be brought within the compaſs of geometry becauſe its ordinates could not be found, as they

they depended upon finding of a ſtreight line equal to the circumference of a circle (ſee Owen's Dictonary, the articles polygon and cycloid); but now from hence it will be very eaſy for any geometrician to lay down the cycloid by points geometrically, which never could be effected before.

The triſection of angles geometrically conſiſts of two general methods; the firſt of which will triſect any angle either right, obtuſe, or acute, as far as 120 degrees, or the double radius; the ſecond will triſect any obtuſe angle that exceeds 120°, even to the ſemicircle.

Firſt, let the angle *AQB*, fig. 5th, be given to be triſected; firſt draw the chord *AB*, produced to *m* and *n*, and divide it in three equal parts *Ao*, *op* and *pB*; make the chord *AB* radius, and ſet it off from *o* to *m* and from *p* to *n*; and on the points *m* and *n* draw the arches *pq* and *or*; join the points *Qq* and *Qr*. I ſay, the angle *AQB* is triſected, for I ſay, angle *AQr* = angle *rQq* = angle *qQB*.

DEMONSTRATION.

But if that ſhould be denied, let the angle *rQ*q + angle *qQB* be conſidered as one angle, and upon the points *r* and *B* biſect it, and it will always be found that the biſecting point will paſs through the triſecting point, and the biſecting line will be coincident with the triſecting line; therefore the angle *rQq* = the angle *qQB*; and alſo, after the ſame manner, may the angle *AQr* be proved equal to angle

angle *r Q q*, therefore the angle *A Q B* is trisected. *Q. E. D.*

Second, let the ∠ *A Q D* be given to be trisected; draw the chord *A D* produced in *g* and *b*, and bisect it in *d*; then take the chord of half the angle (*i. e.*) *A d*, and set it off from *s* to *g*, and from *s* to *b*; then on the points *s* and *g* draw the pricked arches at *t*; also on *s* and *b* draw the pricked arches at *c*, and join the points *Q t* and *Q c*. I say, the angle *A Q D* is trisected; for the same demonstration will prove the truth of it.

Also thirdly, if the acute angle *B Q C* were given to be trisected, draw the chord of the angle *B C*, and proceed according to the first method, and it will be trisected; for that method will trisect any angle right, obtuse, or acute, as far as 120°.

Now to know when your angle is greater or less than 120°, observe the following

RULE.

Take the radius of the angle and apply it to the arch thereof two times, and if twice the radius exceed the angle, use the first method, and if it fall short of the angle, use the second.

The trisecting of angles, geometrically, may be of great use for the raising of polygons in circles; as for example, a polygon of 36 sides may be raised in a circle, by trisecting the four right angles, and so trisecting them again; by which a map of the world may be very readily laid down, and so render the tri-

trisecting compasses of no use; I say, that by trisection and bisection, almost any polygon may be raised in a circle without any mechanical assistance. It may also be of good service to all those who deal in tooth and pinion, and cog and rung, as clock-maker and mill-wrights, &c.

[illegible] By [illegible]

A

New and Eafy Method

Of finding out the

LONGITUDE at SEA

[illegible]

A

New and Easy METHOD

Of finding out the

LONGITUDE *at* SEA.

EPISTLE DEDICATORY.

TO

The Worſhipful the MASTER, WARDENS, and BRETHREN of the Ancient and Honourable TRINITY HOUSE of Newcaſtle upon Tyne.

Gentlemen,

THAT this infant ſcheme of the new and eaſy Method of gaining the Longitude of a Ship at ſea, nurſed up under the influence of your power, may arrive to the ſtature and perfection wiſhed-for, and ſo complete Navigation in its utmoſt extent; which if it does, will be entirely owing to your approbation and protection, for which I ſhall think myſelf happy enough in aſſuring you, that I am, with the trueſt eſteem,

Gentlemen,

Your moſt obedient,

Humble Servant,

JAMES LATIMER.

A

A new and eaſy Method of finding the LONGITUDE *at Sea.*

I Shall here lay down an eaſy and exact method of gaining the Longitude of a ſhip at ſea, and that in ſuch a familiar and plain manner as to render the whole intelligible to the meaneſt capacity; ſo that any common mariner may, with ſuch helps as I ſhall point out to him, take an obſervation of longitude with as much eaſe and exactneſs as he can take an obſervation of latitude.

And this method is briefly thus: Let him provide himſelf with a ſemi-circle of 30 * inches radius, graduated into 180 degrees with a double radius of theſe gradations, and let every degree be divided into minutes; and alſo with an ephemeris of the ſun and moons diſtance from the zodiac, ſuch ephemeriſes are always calculated for the meridian of the metropolis of the country, as London. Now, in order to make uſe of this inſtrument and the ephemeris to advantage, it is neceſſary to premiſe the following preliminaries.

Firſt, that the moon's mean motion in the zodiac, for one day, is about 12·1907 deg. or 12° 11′ 26″, and her hourly motion 30½ minutes nearly; but, in your ephemeris, you have her place in the zodiac, which is her longitude from the ſun, calculated for the meridian of London at noon; therefore, in order to take an obſervation to advantage,

 you

* See the advertiſement at the end of this method.

you must first find the latitude your ship is in, and then, according to the day of the month, find the sun's rising or setting for that day, according as you judge yourself east or west from the meridian of London, which is a very easy problem in astronomy; and having got the sun's rising or setting for the day of the month, according to the latitude your ship is in, then look into your ephemeris for the same day of the month, and there find the moon's place in the zodiac, and also the sun's place in the ecliptic: if it is before the full, the moon's distance from the sun will be less than six signs or 180 degrees, which will suit for evening observations, and after the full for morning observations.

Second, if it is before the full, and you judge your ship to the westward of the meridian of London, then the sun will be longer in setting where the ship is than at any part of the globe that is under the meridian of London; but if she is to the eastward of the meridian of London, then the sun will set sooner where the ship is than at any part of the globe that is under the meridian of London.

These things being premised, let us suppose ourselves near the banks of Newfoundland, being in the same parallel of latitude with London, 51° 30′, March 20. The sun is then in the first degree of Aries; and suppose you take an evening observation by the semicircle, and find the moon's distance from the sun to be 90° at his setting; and looking in the tables, we find the moon's distance from the

the ſun, when ſetting at London, to be only 87° 58′; ſo that the difference betwixt the obſervation and the tables is 2° 2′, and ſo much the moon has travelled betwixt the ſun's ſetting at London and the time of his ſetting where the ſhip is. How to know what ſpace of time the moon took in travelling theſe 2° 2′, (which is the difference of her diſtance from the ſun between the obſervation and and the tables) divide the 2° 2′ by her hourly motion 30.5′, and it will give the time four hours; which ſhews that the ſun is four hours longer in ſetting where the ſhip is than at London, and by allowing 15° for every hour of time, we have the ſhip's longitude weſt from London 60°.

Now, in order to take a morning obſervation, the moon being paſt the full, let us ſuppoſe her 22 days old, entering into her laſt quarter; and at ſun riſing with your ſemi-circle, taking the Moons diſtance from the ſun, which will be found only 87° 58′, which is leſs than the diſtance at London in the tables by 2° 2′, which divided by her hourly motion 30.5′ will give four hours, the time the ſun is longer in riſing where the ſhip is than at London; and allowing for every hour of time 15°, you'll have 60°, the ſhips longitude weſt from London, the ſame as in the evening obſervation.

Now, if the ſhip was eaſt of the meridian of London, it would only be the reverſe of this, the Sun riſing or ſetting ſooner where the ſhip is than at London; ſo the difference of your

tables

tables and semi-circle, converted into time and measure, will give the ship's longitude east from London.

If a proper set of tables were made of the moon's longitude from the sun's, and calculated from every degree of latitude, betwixt the artic and antartic circles, for the meridian of London, and tables of the sun's rising and setting, corresponding thereto; I say, with such a set of tables and semi-circles, as shall be pointed out, that any mariner may gain the longitude of his ship to half a degree, which is sometimes reckoned not a bad observation for latitude.

The semi-circle that shall be hereafter advertised, tho' of but six or eight inches radius, will take an angle very accurately to 30 seconds of a minute; and therefore the moon's distance from the sun may be taken to the greatest exactness (*i. e.*) to less than a minute if needful; for the moon's hourly motion $30'$ $30''$ = an hour = 15° of longitude, $15'$ $15''$ = half an hour = $7\frac{1}{2}^\circ$; $7'$ $37''$ = $\frac{1}{4}$ of an hour = $3\frac{1}{4}^\circ$; $3'$ $48''$ = $\frac{1}{8}$ of an hour = $1\frac{1}{4}^\circ$; $1'$ $54''$ = $\frac{1}{16}$ of an hour = $27'$ of a degree: for 360° divided by 24 hours = 15°.

A TABLE *of the* Moons *mean distance from the* Sun *in* Degrees, Minutes, *and* Seconds, *to every six hours of her age, acoording to her syndical motion.*

da.	ho.	deg.	min.	sec.	da.	ho.	sec.	deg.	min.	sec	da.	ho.	sec.	deg.	min.	sec.
0	6	3	2	51	10	6	0	124	57	19	20	0	0	116	11	5
0	12	6	5	43	10	12	0	128	0	10	20	6	0	113	8	13
0	18	9	8	35	10	18	0	131	3	2	20	12	0	110	5	22
1	0	12	11	27	11	0	0	134	5	54	20	18	0	107	2	30
1	6	15	14	18	11	6	0	137	8	45	21	0	0	103	59	38
1	12	18	17	10	11	12	0	140	11	37	21	6	0	100	56	47
1	18	21	20	2	11	18	0	143	14	29	21	12	0	97	53	55
2	0	24	22	53	12	0	0	146	17	20	21	18	0	94	51	3
2	6	27	25	45	12	6	0	149	20	12	22	0	0	91	48	12
2	12	30	28	37	12	12	0	152	23	4	22	6	0	88	45	20
2	18	33	31	28	12	18	0	155	25	56	22	12	0	85	42	28
3	0	36	34	20	13	0	0	158	28	47	22	18	0	82	39	37
3	6	39	37	12	13	6	0	161	31	39	23	0	0	79	36	45
3	12	42	40	3	13	12	0	164	34	31	23	6	0	76	33	53
3	18	45	42	55	13	18	0	167	37	22	23	12	0	73	31	2
4	0	48	45	47	14	0	0	170	40	14	23	18	0	70	28	10
4	6	51	48	38	14	6	0	173	43	6	24	0	0	67	25	18
4	12	54	51	30	14	12	0	176	45	57	24	6	0	64	22	26
4	18	57	54	22	14	18	0	179	48	49	24	12	0	61	19	35
5	0	60	57	14	14	18	22	179	59	59	24	18	0	58	16	43
5	6	64	0	5	15	0	0	177	8	19	25	0	0	55	13	51
5	12	67	2	57	15	6	0	174	5	27	25	6	0	52	11	0
5	18	70	5	48	15	12	0	171	2	35	25	12	0	49	8	8
6	0	73	8	40	15	18	0	167	59	44	25	18	0	46	5	16
6	6	76	11	32	16	0	0	164	56	52	26	0	0	43	2	25
6	12	79	14	24	16	6	0	161	54	0	26	6	0	39	59	33
6	18	82	17	15	16	12	0	158	51	9	26	12	0	36	56	41
7	0	85	20	7	16	18	0	155	48	17	26	18	0	33	53	50
7	6	88	22	59	17	0	0	152	45	25	27	0	0	30	50	58
7	12	91	25	50	17	6	0	149	42	34	27	6	0	27	48	6
7	18	94	28	42	17	12	0	146	39	42	27	12	0	24	25	15
8	0	97	31	34	17	18	0	143	36	50	27	18	0	21	42	23
8	6	100	34	25	18	0	0	140	33	59	28	0	0	18	39	31
8	12	103	37	17	18	6	0	137	31	7	28	6	0	15	36	4
8	18	106	40	9	18	12	0	134	28	15	28	12	0	12	33	48
9	0	109	43	0	18	18	0	131	25	23	28	18	0	9	30	56
9	6	112	45	52	19	0	0	128	22	32	29	0	0	6	28	4
9	12	115	48	44	19	6	0	125	19	40	29	6	0	3	25	13
9	18	118	51	35	19	12	0	122	16	48	29	12	0	0	22	21
10	0	121	54	27	19	18	0	119	13	57	29	12	44	0	0	0

The uſe of this table, of the moon's daily and hourly mean motion, is for the ready adjuſting of the inſtruments, before they come to obſerve and take her diſtance from the ſun, which will always be within a degree of the truth; and by turning the index by the endleſs ſcrew backwards or forwards, the moon's true diſtance may be taken in leſs than a minute's time.

I ſhall here give another example or two, to ſhew the uſe of Turnbull's ſemi-circle, when applied to this method.

EXAMPLE 1ſt, *Moon four days old.*

Suppoſe now your ſhip ſomewhere in the Atlantic ocean, and you had a good obſervation for latitude, and you would be glad to know your longitude by obſervation alſo.

Nigh ſun-ſetting, if the night is clear, rectify your inſtrument, by turning the index of the mirror to the hour of the moon's age as near as poſſible, then ſcrew it faſt; alſo turn the long index to the degree and minute of her diſtance, anſwering to her age in the table, (laſt page) and your inſtrument is rectified as near as the nature of it will admit.

Then, when the ſun's under limb begins to touch the horizon, hold up your inſtrument ſo as the plane thereof may paſs through the center of both luminaries, making the ſun to ſhine through the right-hand ſight if the ſun is ſouth of you, but through the left-hand one if he is north of you: Thus, ſet your eye to the ſight in the end of the long index, turning

ing it about until you can see the moon through the clear part of the mirror, and the image of the sun in the silvered part; bring their nighest limbs exactly in contact, then observe the degrees, minutes, and seconds, which suppose 46° 4′ 30″; to which add the sum of their semi-diameters at that time, suppose 31′, and also the sun's horizontal refraction 33′, and the sum is 47° 8′ 30″, which call the observed distance.

Then look in your ephemeris for the moon's distance from the sun, at his setting, for that day of her age, and that latitude under the meridian of London; which suppose 45° 52′ 50″. This taken from the observed distance, leaves 1° 15′ 40″, the distance gone by the moon since the sun set under the meridian of London. This divided by 30′ 28″, her hourly motion, quotes 2·48358 hours. Then say, if 1 hour give 15°, what will * 2·48358 hours give? Ans. 37° 15′, the longitude the ship is in west, when the observed distance is greater than the ephemeris; but east, when it is less.

EXAMPLE 2*d*, *Moon* 8½ *days old.*

Suppose a ship somewhere beyond Madagascar in the Indian ocean, on a voyage to the East Indies, and you have secured your latitude by observation.

Rectify

* I would advise the Practitioner to bring this time always out in decimals; for then multiplying by 15, gives the longitude less liable to error, than when it is reduced to degrees, minutes, and seconds.

Rectify your instrument as before taught, then observe at sun setting, and suppose your observed distance to be 104° 42′ 40″; then look in your ephemeris for the moon's distance from the sun at his setting under the meridian of London, &c. which suppose 106° 56′ 20″, the observed distance taken from this, leaves 2° 13′ 40″, the distance the moon will go before the sun set at London; which divide by 30′ 28″, quotes 4·387 hours; and this time multiplied by 15°, gives 65° 48′, the longitude the ship is in east.

N. B. In an evening observation, the refraction of the sun and the parallax of the moon is to be added to their distance by observation; and, in the morning observation, to be subtracted from the above distance.

F I N I S.

E R R A T A.

The Trisection of Angles geometrically begins at page 25, line 8, instead of running on with the subject before.

It also continues to page 27.

ADVERTISEMENT.

MY ingenious friend Mr Robert Turnbull, has invented a ſemi-circle of ſix inches radius that will take the diſtance of the Moon from the Sun by reflection, even to leſs than 30 ſeconds of a minute, which is more than is needful in this caſe; for if it be taken to a minute, it will anſwer the end to all intents and purpoſes.

N. B. This inſtrument was perfected, and compleated by Henry Latimer, clock and watch-maker in Gateſhead; by applying of a wheel, and an endleſs ſcrew to it, which keeps it ſo firm that it cannot ſhake in the leaſt; and moves it with the greateſt nicety.